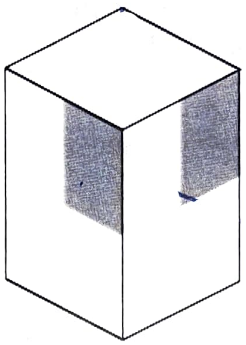
Exam Symmetry in Physics

Date	December 4, 2025
Time	17:00 - 19:00
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- Raise your hand for more paper
- You are not allowed to use the lecture notes, nor other notes or books
- Use of a calculator is allowed
- All subquestions (a, b, c) of the three exercises have equal weight
- Illegible answers will be not be graded
- Explain or justify your answers
- Contradictory answers will be considered invalid
- Good luck!

Exercise 1

Consider the square block depicted in the figure below:



The base of the block is a regular square. The rectangular sides all have the exact same pattern where the upper right quarter is grey and the rest is white.

(a) Identify all symmetry transformations (rotations and reflections) that leave this block invariant and call the group that they form G_{SB}. Also identify all conjugacy classes and briefly specify the conjugating elements, e.g. by writing $g \overset{h}{\sim} g'$ to mean $g = hg'h^{-1}$. You are allowed to use geometric arguments.

(b) Construct the character table of G_{SB} and explain how the entries are obtained.

(c) Show whether G_{SB} allows for an invariant vector, an invariant axial vector, or neither.

Exercise 2

Consider the group $O(3)$ of real orthogonal 3×3 matrices and its action on the spatial part of the electromagnetic field strength tensor

$$F_{ij} = \nabla_i A_j - \nabla_j A_i,$$

where i, j can take the values 1, 2, and 3, and A_i denotes the vector potential.

(a) Show that F_{ij} remains antisymmetric under rotations and specify under which irrep of $SO(3)$ it transforms, where $SO(3)$ denotes the rotational subgroup of $O(3)$. The irrep may be specified by the dimension $d = 2l + 1$.

(b) Determine how F_{ij} transforms under reflections in the origin (space inversion).

(c) Explain how the magnetic field $B_i = \frac{1}{2}\epsilon_{ijk}F_{jk}$, or equivalently $\vec{B} = \vec{\nabla} \times \vec{A}$, transforms under $O(3)$ transformations.

Exercise 3

Consider the group $SU(2)$ of unitary 2×2 matrices with determinant equal to 1. Consider its action on the states $|j, m\rangle$ through the operator

$$U(\theta, \hat{n}) = \exp\left(\frac{i}{\hbar}\theta\, \hat{n} \cdot \vec{J}\right),$$

where $\vec{J}$ denotes the angular momentum operator.

(a) Explain why the operator $U(\theta, \hat{n})$ can not change the quantum number j.

(b) Explain why the matrix representation of $U(\theta, \hat{n})$ for the specific case $\hat{n} = \hat{z}$ is a diagonal matrix when acting on states $|j, m\rangle$ and what is its dimension?

(c) Explain which irreps of $SU(2)$ arise in the addition of angular momentum of a system with $j_1 = \frac{5}{2}$ and a system with $j_2 = \frac{3}{2}$.